MCB-003-1162003 Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

April / May - 2018

Mathematics : CMT - 2003

(Topology – II)

Faculty Code : 003
Subject Code : 1162003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions in this paper.
(2) Each question carries 14 marks.
(3) All questions are compulsory.

1 Fill in the blanks : (Each question carries two marks) **14**

(a) In a Hausdorff space every singleton subset is __________

(b) Every closed subspace of a compact space is __________

(c) The one point compactification of a locally compact, non-compact hausdorff space is __________ and __________

(d) A closed subspace of any complete metric space is __________

(e) Tietz's extension theorem is equivalent to the separation axiom __________

(f) A closed and bounded subset of $\mathbb{Q}$ need not be __________

(g) An infinite set with co-finite topology satisfies __________ separation axiom but does not satisfy __________ separation axiom.

2 Attempt any **two** of the following : **14**

(a) Prove that any open subspace of a locally compact Hausdorff space is locally compact.

(b) State and prove Lebesgue's covering lemma.

(c) Prove that

(i) Every subspace of a $T_1 - space$ is $T_1 - space$.

(ii) Suppose $X \times Y$ is hausdorff. Prove that X and Y both are hausdorff.

3 All are compulsory : **14**

(a) Prove that any compact hausdorff space is regular. **6**

(b) Prove that a T_1 space X is regular if and only if for every open set U and $x \in U$ there is an open set V such that $x \in V \subset \overline{V} \subset U$. **4**

(c) Prove that $(\mathbb{R}, d)$ is a complete metric space. **4**

OR

3 All are compulsory : **14**

(a) Prove that every sequentially metric space is compact. **6**

(b) Give an example of an infinite topological space which is not compact. **4**

(c) Prove that a T_1 space X is normal if and only if for each closed set A and an open set U with $A \subset U$ there is a closed set V that $A \subset V \subset \overline{V} \subset U$. **4**

4 Attempt any **two** of the following : **14**

(a) State Tube Lemma and then prove that $X \times Y$ is compact if both X and Y are compact.

(b) Prove that $C(X, Y)$ and $B(X, Y)$ are closed subspaces of the space Y^X (with the topology induced from uniform metric).

(c) Prove that $\mathbb{R}^n$ is a complete metric space.

5 Do as directed : (Each question carries two marks) **14**

(a) Give the two subsets of $\mathbb{R}$ (the set of real numbers with standard topology) such that one is closed but not bounded and the other is bounded but not closed.

(b) Give the definition of a limit point compact space.

(c) State (i) Urysohn's lemma. (ii) Tietz's extension theorem.

(d) Give an example of uncountable subset of $\mathbb{R}$ which is not locally compact.

(e) Determine if the set $\mathbb{R}-\{0\}$ is a complete subspace of $\mathbb{R}$ or not?

(f) $A=\left\{1,\frac{1}{2},\frac{1}{3},........,\frac{1}{n}\right\} U \{0\}$. Is A a locally compact subset of $\mathbb{R}$? Give reasons for your answer.

(g) Give an example of compact metric space which is uncountable.